Assumption-Based Approaches to Reasoning with Priorities.

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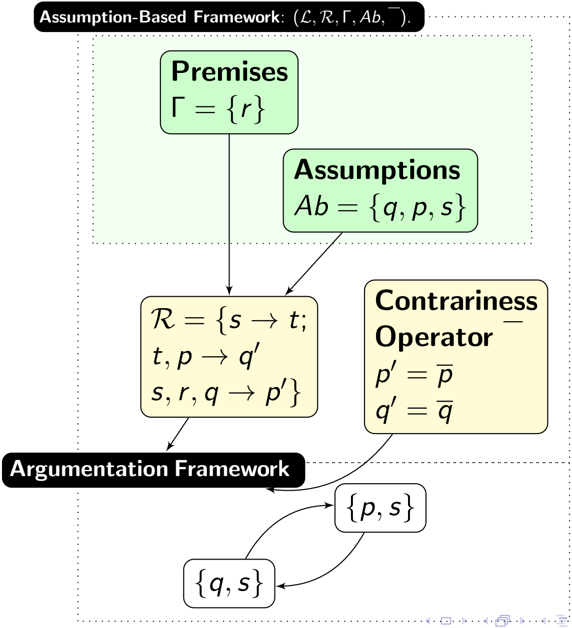
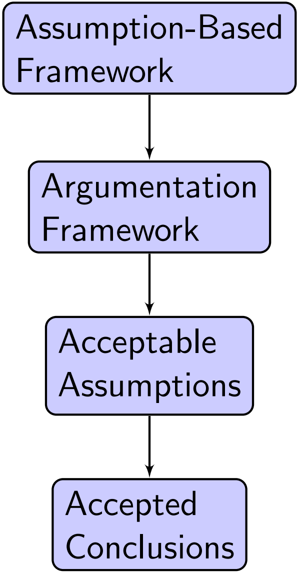
The Plan

1. Assumption-Based Argumentation
2. The Relation between *d*- and *r*-defeat
3. Translating ABA^d into ABA^f.
4. Translating ABA^r into ABA^d
   - $\mathsf{ABA}^r_\wedge$ and $\mathsf{ABA}^d_\wedge$
   - $\mathsf{ABA}^r_\wedge$ to $\mathsf{ABA}^d_\wedge$
5. Outlook

Assumption-Based Argumentation



The Argumentation Pipeline.



Assumptions-based Frameworks

Definition (Assumption-based framework)

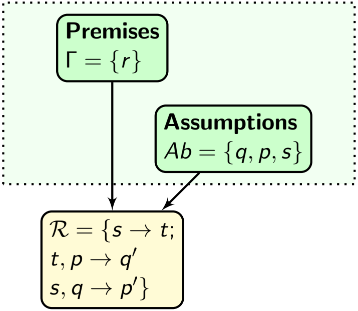
An assumption-based framework is a tuple $\mathbf{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, \leq)$ *where:*

- $\mathcal{L}$ *is a formal language*
- $\mathcal{R}$ *is a set of rules*
- $\emptyset \neq Ab \subseteq \mathcal{L}$ *is the set of candidate assumptions.*
- $\overline{} : Ab \to \wp(\mathcal{L})$ *is a contrariness operator.*
- $\leq$ *is a total order over the assumptions.*

Flat Frameworks

We will additionally assume that frameworks are flat, i.e.
$A_1, \ldots, A_n \to A \notin \mathcal{R}$ for $A \in Ab$.

Deductive System $(\mathcal{L}, \mathcal{R})$



Example

- $\{s\} \vdash_{\mathcal{R}} t$
- $\{s, p\} \vdash_{\mathcal{R}} q'$

Definition ($\mathcal{R}$-deduction)

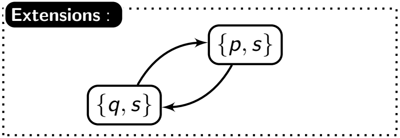
An $\mathcal{R}$-deduction from Δ of A, written $\Delta \vdash_{\mathcal{R}} A$, is a finite tree where

1. *the root is A,*
2. *the leaves are either empty nodes or elements from Δ,*
3. *the children of non-leaf nodes are the conclusions of rules in $\mathcal{R}$ whose antecedent correspond to their children,*
4. *Δ is the set of all $A \in Ab$ that occur as leaves in the tree.*

Attacks

Example

- $Ab = \{p, q, s\}$.
- $\mathcal{R} = \{q \to \overline{p}; p \to \overline{q}\}$
- $\{q\} \vdash_{\mathcal{R}} \overline{p}$.
- $\{p\} \vdash_{\mathcal{R}} \overline{q}$.



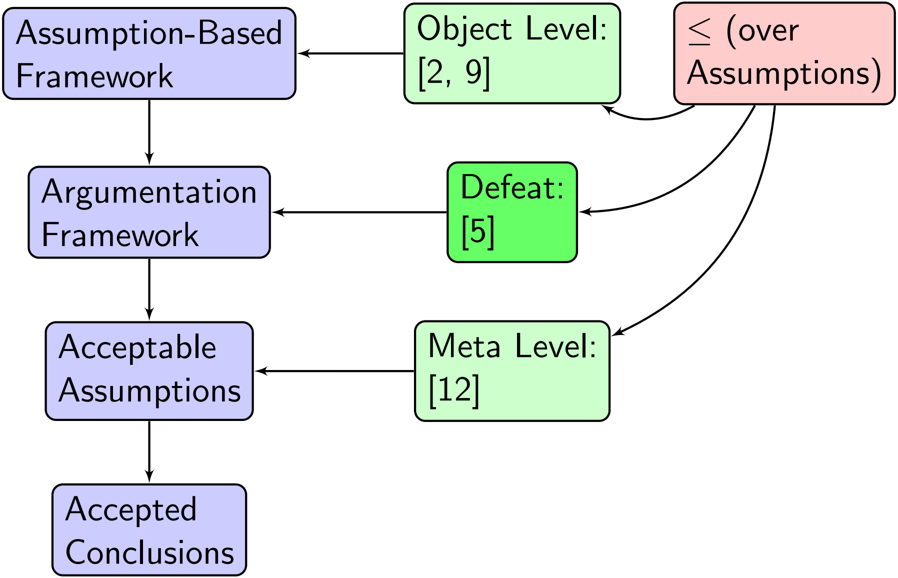
Definition (Attacks)

Given an assumption-based framework $\mathbf{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{})$, *a set of assumptions* $\Delta \subseteq Ab$:

- Δ *attacks an assumption* $A \in Ab$ *iff* $\Delta' \vdash_{\mathcal{R}} \overline{A}$ *for some* $\Delta' \subseteq \Delta$.
- Δ *attacks a set of assumptions* $\Theta \subseteq Ab$ *iff* Δ *attacks some* $A \in \Theta$.

We'll denote attack with the symbol $\hookrightarrow_f$.

The Argumentation Pipeline: where do Priorities come in?.



Comparing Sets of Assumptions

Definition (Lifting $\leq$)

Given an assumption-based framework $\textbf{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, \leq)$ *and* $\Delta \subseteq Ab$, *we define:*

- $\emptyset \not< A$ *for any* $A \in Ab$ *and*
- $\Delta < A$ *if* $B < A$ *where* $\{B\} = \min(\Delta)$.

From Attack to Defeat

Definition (Attack, defeat, reverse defeat)

Given an assumption-based framework $\mathbf{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, \leq)$ *is a set of assumptions* $\Delta \subseteq Ab$ *and an assumption* $A \in Ab$*, we say that:*

- Δ *d-defeats* A *iff there is a* $\Delta' \subseteq \Delta$ *s.t.* $\Delta' \vdash_{\mathcal{R}} B$ *for some* $B \in \overline{A}$ *and* $\Delta' \not< A$*.*
- Δ *d-defeats* Θ *if* Δ *d-defeats some* $A \in \Theta$*.*
- Δ *r-defeats* $\Theta \subseteq Ab$ *iff either*
  - Δ *d-defeats* Θ*, or*
  - *there is a* $\Theta' \subseteq \Theta$ *s.t.* $\Theta' \vdash_{\mathcal{R}} B$ *for some* $B \in \overline{A}$*,* $A \in \Delta$ *and* $A > \Theta'$

We will also denote d-defeat and r-defeat with, respectively, the symbols $\hookrightarrow_d$ and $\hookrightarrow_r$.

Example

- Björn wants to go out with his friends Agnetha (A), Benny (B) and Frida (F).
- If Benny is together with Agnetha, he doesn't want to go out with Frida ($A, B \rightarrow \overline{F}$).
- Björn likes Benny more then Agnetha ($A < B$).
- Björn likes Frida more then Benny ($B < F$).

$$\{A, B\} \hookrightarrow_f \{F\}$$
$$\{F\} \hookrightarrow_r \{A, B\} \quad \{A, B\} \not\hookrightarrow_d \{F\}$$

Conflict-Free Sets of Assumptions

Example (for f-defeat)

- $Ab = \{p, q, s\}$.
- $\mathcal{R} = \{q \to \overline{p}; p \to \overline{q}\}$
- $\{q\} \vdash_{\mathcal{R}} \overline{p}$.
- $\{p\} \vdash_{\mathcal{R}} \overline{q}$.
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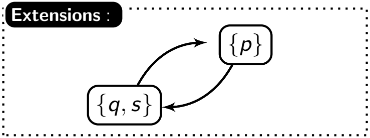

Definition (Argumentation semantics)

Where $\Delta \subseteq Ab$ *and* $x \in \{d, r, f\}$, Δ *is:*

Conflict-Free Sets of Assumptions

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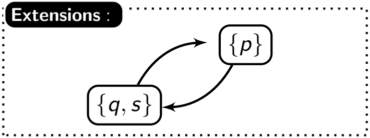

Definition (Argumentation semantics)

Where $\Delta \subseteq Ab$ and $x \in \{d, r, f\}$, Δ is:

- x-**conflict-free** *iff for every* $\Delta' \cup \Delta'' \subseteq \Delta$, $\Delta' \not\rightarrow_x \Delta''$.

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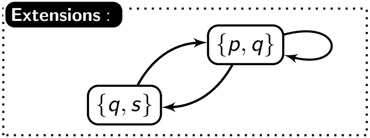

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Admissibility Semantics

Example

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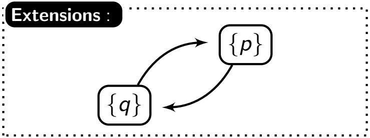

Definition (Argumentation semantics)

Where $\Delta \subseteq Ab$ and $x \in \{d, r, f\}$, Δ is:

- *is x-**admissible** iff it is x-conflict-free and for each set of assumptions $\Theta \subseteq Ab$, if $\Theta \hookrightarrow_x \Delta$, then $\Delta \hookrightarrow_x \Theta$.*

Admissibility Semantics

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- $Ab = \{p, q, s\}$.
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- $\{p\} \vdash_{\mathcal{R}} \overline{q}$.

Extensions :

$$\begin{tikzcd} & \boxed{\{p, s\}} \arrow[dl, bend left] \\ \boxed{\{q, s\}} \arrow[ur, bend left] & \end{tikzcd}$$

Definition (Argumentation semantics)

Where $\Delta \subseteq Ab$ and $x \in \{d, r, f\}$, Δ is:

- *is x-**admissible** iff it is x-conflict-free and for each set of assumptions $\Theta \subseteq Ab$, if $\Theta \hookrightarrow_x \Delta$, then $\Delta \hookrightarrow_x \Theta$.*
- *is x-**preferred** iff it is maximally (w.r.t. set inclusion) x-admissible.*

The Relation between *d*- and *r*-defeat

$$\{A, B\} \hookrightarrow_f \{F\}$$
$$\{F\} \hookrightarrow_r \{A, B\} \quad \{A, B\} \not\hookrightarrow_d \{F\}$$

Definition (Contraposition [10])

$\mathbf{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, \mathrm{Val}, \leq)$ *is closed under contraposition if for every* $\Delta \subseteq Ab$*:*

$$\textit{if } \Delta \vdash_{\mathcal{R}} C \textit{ for some } C \in \overline{A}$$

then for every $B \in \Delta$ *it holds that*

$$(\{A\} \cup \Delta) \setminus \{B\} \vdash_{\mathcal{R}} D \textit{ for some } D \in \overline{B}.$$

The Relation between d- and r-defeat

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Conjecture

- r-defeat seems to be a kind of contraposition.
- So perhaps if **ABF** is closed under contraposition, r-defeat and d-defeat coincide?

Well. . . [10]

Let $Ab = \{a, b, c, d\}$ and $d > b$, Let $\widehat{x} \in \overline{x}$ for every $x \in \{a, b, c, d\}$, and

$$\mathcal{R} = \begin{Bmatrix} b, c \to \widehat{d} & b, d \to \widehat{c} & c, d \to \widehat{b} & b \to \widehat{b} \\ b, c \to \widehat{a} & a, c \to \widehat{b} & a, b \to \widehat{c} & c \to \widehat{c} \end{Bmatrix}$$

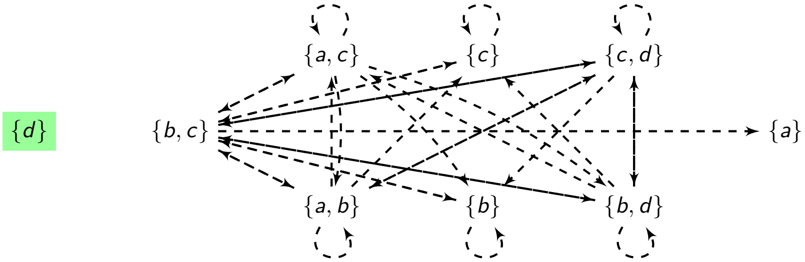

Figure: Direct defeats are represented by dashed arrow whereas *r*-defeats are represented by dotted-arrows.

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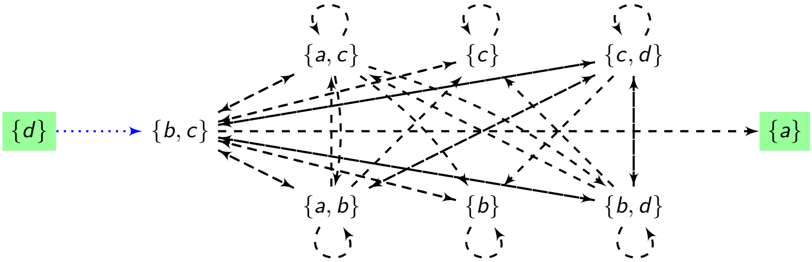

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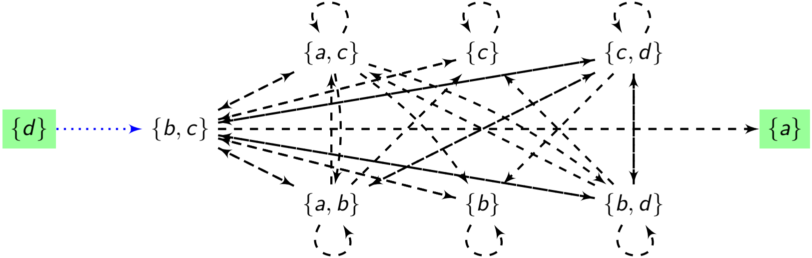


Figure: Direct defeats are represented by dashed arrow whereas r-defeats are represented by dotted-arrows.

Note the large ammount of self-defeating sets of assumptions.

The Relation between d- and r-defeat

Definition (Cycle-Freeness)

ABF $= (\mathcal{L}, \mathcal{R}, Ab, \overline{}, \mathrm{Val}, \leq)$ *is cycle-free if for every* $\Delta \subseteq Ab$*: if* $A \in \Delta$ *then:*

$$\Delta \nvdash_{\mathcal{R}} B \quad \textit{for any } B \in \overline{A}.$$

Theorem

If **ABF** *is closed under contraposition and cycle-free then:*
Δ *is* d*-preferred iff* Δ *is* r*-preferred.*

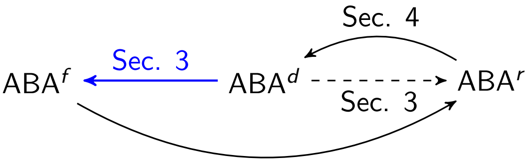
Cycle-Free **ABF**s

- Cycle-Free **ABF**s have not been studied in the literature yet.
- Seems a valuabe concept (e.g. for studying crash-resistance in ABA).
- However, since their behaviour is not well-known, we provide translations between ABA^d and ABA^r .

The Plan

$$\begin{tikzcd}[column sep=huge]
\mathrm{ABA}^f \arrow[rr, bend right, ""] & \mathrm{ABA}^d \arrow[l, "\text{Sec. 3}"'] \arrow[r, dashed, "\text{Sec. 3}"'] & \mathrm{ABA}^r \arrow[l, bend right, "\text{Sec. 4}"']
\end{tikzcd}$$

From ABA^d to ABA^f.



Example

Example

- *Let* $Ab = \{p, q\}$, $p < q$,
- $\mathcal{R} = \{q \rightarrow p'\}$ *and*
- $p' = \overline{p}$.

ABA^d

- $\{q\} \vdash_{\mathcal{R}} p'$ and
- $p < q$.
- Consequently, $\{q\} \hookrightarrow_d p$

- For every element of the language $A \in \mathcal{L}$ we now have elements A^i.
- The superscripts are used to express priorities in the object language.
- Rules are translated in such a way that the superscripts are carried over in the right way.

Getting a grip on $\leq$

Given $(\mathcal{L}, \mathcal{R}, Ab, \overline{}, \leq)$ we suppose that:

- there is a totally ordered set $(\mathrm{Val}, \preceq)$ and
- a function $f : Ab \rightarrow \mathrm{Val}$ such that:
  - $a \leq b$ iff $f(a) \preceq f(b)$
- We will further expand the set Val with a maximum element ω, i.e. with $\alpha \prec \omega$ for all $\alpha \in \mathrm{Val}$, and (abusing notation) refer to the resulting set $\mathrm{Val} \cup \{\omega\}$ simply as Val.

Translation: Formal

Definition (Translation τ)

We translate a given framework $\mathbf{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, \leq)$ *into the following framework* $\tau(\mathbf{ABF}) = (\tau(\mathcal{L}), \tau(\mathcal{R}), \tau(Ab), \overline{}, (\tau(Ab) \times \tau(Ab)))$*:*

- $\tau(\mathcal{L}) = \{A^\alpha \mid A \in \mathcal{L}, \alpha \in \mathrm{Val}\}$
- *where* $\rightarrow A \in \mathcal{R}$, $\tau(\rightarrow A) = \rightarrow A^\omega$.
- *where* $A_1, \ldots, A_n \rightarrow A \in \mathcal{R}$ *and* $\min(\{\alpha_1, \ldots, \alpha_n\}) = \{\alpha\}$,

$$\tau(A_1, \ldots, A_n \rightarrow A) = {A_1}^{\alpha_1}, \ldots, {A_n}^{\alpha_n} \rightarrow A^\alpha$$

- $\tau(\mathcal{R}) = \{\tau(r) \mid r \in \mathcal{R}\}$.
- $\tau(Ab) = \{A^{f(A)} \mid A \in Ab\}$
- $A^\alpha \in \overline{B^\beta}$ *iff* $A \in \overline{B}$ *and* $\alpha \not< \beta$.

Example

Example

Let $Ab = \{p, q\}$, $p < q$ *and* $\mathcal{R} = \{q \to p'\}$ *and* $p' = \overline{p}$.

- $\mathrm{Val} = \{1, 2\}$
- $f(p) = 1$
- $f(q) = 2$
- $\tau(Ab) = \{p^1, q^2\}$.
- $\tau(\mathcal{R}) \ni q^2 \to (p')^2$
- $p'^2 \in \overline{p^1}$.

ABA^d

- $\{q\} \vdash_{\mathcal{R}} p'$ and
- $p < q$.
- Consequently, $\{q\} \hookrightarrow_d p$

ABA^f

- $\{q^2\} \vdash_{\mathcal{R}} (p')^2$ and
- Consequently, $\{q^2\} \hookrightarrow_f p^1$

Adequacy

Theorem

Given an assumption-based framework $\mathbf{ABF}$*:*
Δ *is d-preferred (in* $\mathbf{ABF}$*) iff* $\tau(\Delta)$ *is* f*-preferred (in* $\tau(\mathbf{ABF})$*)*

From ABA^r to ABA^d.

$$
\begin{tikzcd}
\mathrm{ABA}^f \arrow[rr, bend right] & \mathrm{ABA}^d \arrow[l, "\text{Sec. 3}"'] \arrow[r, dashed, "\text{Sec. 3}"'] & \mathrm{ABA}^r \arrow[l, bend right, "\text{Sec. 4}"']
\end{tikzcd}
$$

- $Ab = \{A, B, F\}$
- $\mathcal{R} = \{A, B \to \overline{F}\}$
- $A < B < F$.

$$\{A, B\} \hookrightarrow_f \{F\}$$
$$\{F\} \hookrightarrow_r \{A, B\} \quad \{A, B\} \not\hookrightarrow_d \{F\}$$
$$\{F\} \not\hookrightarrow_r \{A\} \quad \{F\} \not\hookrightarrow_r \{B\}$$

$$\begin{tikzcd}
\mathrm{ABA}^d_\wedge \arrow[r, bend right] & \mathrm{ABA}^r_\wedge \arrow[l, "\text{Section 4.2}"'] \arrow[r, leftrightarrow, "\text{Section 4.1}"] & \mathrm{ABA}^r \arrow[ll, dashed, bend right]
\end{tikzcd}$$

Closing Frameworks under Conjunction.

$$\begin{tikzcd}[column sep=huge] \mathrm{ABA}^d_\wedge \arrow[r, bend right] & \mathrm{ABA}^r_\wedge \arrow[l, "\text{Section 4.2}"'] \arrow[r, leftrightarrow, blue, "\text{Section 4.1}"] & \mathrm{ABA}^r \arrow[ll, bend right, dashed] \end{tikzcd}$$

Definition (Conjunction)

Where $\mathbf{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, \leq)$, *we define* $\mathbf{ABF}^{\wedge} = (\mathcal{L}^{\wedge}, \mathcal{R}^{\wedge}, Ab^{\wedge}, \overline{}, \leq)$, *where:*

- $\mathcal{L}^{\wedge} = \{A_1 \wedge \ldots \wedge A_n \mid A_1, \ldots, A_n \in \mathcal{L}, n \in \mathbb{N}\}$.
- $\mathcal{R}^{\wedge}$ *is the smallest set:*
  - *containing* $\mathcal{R}$.
  - *closed under:*
  - *($\wedge$-introduction)* $A_1, \ldots, A_n \rightarrow \bigwedge\{A_1, \ldots, A_n\}$
  - *($\wedge$-elimination)* $\bigwedge \Delta \rightarrow A$ *for all* $A \in \Delta$
- $Ab^{\wedge} = \{\bigwedge \Delta' \mid \Delta' \subseteq_{\text{fin}} Ab\}$

For any $\Delta \subseteq Ab$, *let:*

- $\Delta^{\wedge} = \{\bigwedge \Delta' \mid \Delta' \subseteq_{\text{fin}} \Delta\}$
- $\Delta_{\wedge} = \{A \mid \bigwedge \Delta' \in \Delta, A \in \Delta'\}$.

Definition ($\mathbf{ABF}^{\wedge}$ framework, continued)

Given an $\mathbf{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, \leq)$ *framework, we define* $\mathbf{ABF}^{\wedge} = (\mathcal{L}^{\wedge}, \mathcal{R}^{\wedge}, Ab^{\wedge}, \overline{}, \leq)$ *where* $\mathcal{L}^{\wedge}, \mathcal{R}^{\wedge}$ *and* $Ab^{\wedge}$ *are defined as above, and (abusing notation),*

- $\leq$ *is extended to* $Ab^{\wedge}$ *as follows:*
  - $\min(\Delta) =_{\text{df}} \min(\{f(A) \mid A \in \Delta\})$.
  - *Where* $\Delta \subseteq Ab$, $f(\bigwedge \Delta) =_{\text{df}} \min(\Delta)$.
  - *Where* $\Delta \subseteq Ab^{\wedge}$ *we define* $\min(\Delta) =_{\text{df}} \min(\Delta_{\wedge})$.

Example

Let $Ab = \{A, B, F\}$, $\mathcal{R} = \{A, B \to \overline{F}\}$ *and* $A < B < F$.

- $\mathcal{L}^{\wedge} = \{A, B, F, A \wedge B, B \wedge F, A \wedge F, A \wedge B \wedge F\}$.
- $\mathcal{R}^{\wedge} = \mathcal{R} \cup \{A, B \to A \wedge B; A \wedge B \to A; A \wedge B \to B, \ldots\}$.
- $Ab^{\wedge} = \{A, B, F, A \wedge B, B \wedge F, A \wedge F, A \wedge B \wedge F\}$

Semantics for $ABA_\wedge$

Definition

Given $\mathbf{ABF} = (\mathcal{L}, \mathcal{R}, Ab, \overline{}, \leq)$ $x \in \{d, r, f\}$ *and* $\Delta \subseteq Ab^\wedge$, *let* $\wp^\wedge(\Delta)$ *be the set of all* $\Delta' \subseteq \Delta$ *that are closed under* $\wedge$*-intro and* $\wedge$*-elim.*

- Δ *is* $\wedge$*-closed iff* $\Delta \in \wp^\wedge(Ab)$
- Δ *is* x*-*$\wedge$*-conflict-free iff there are no* $\Delta_1, \Delta_2 \in \wp^\wedge(\Delta)$ *such that* Δ_1 x*-defeats* Δ_2.
- Δ *is* x*-*$\wedge$*-admissible iff it is* x*-*$\wedge$*-conflict-free,* $\wedge$*-closed and for all* $\Theta \in \wp^\wedge(Ab)$ *that* x*-defeat* Δ*, there is a* $\Delta' \in \wp^\wedge(Delta)$ *that* x*-defeats* Θ.

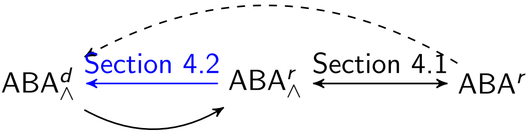
x*-*$\wedge$*-preferred extensions are defined as usual.*

Adequacy

Theorem

Given an assumption-based framework **ABF** *and* $x \in \{d, r\}$*:*
Δ *is* x*-preferred (in* **ABF***) iff* $\Delta^{\wedge}$ *is* x*-*$\wedge$*-preferred (in* $\mathbf{ABF}^{\wedge}$*).*

$\mathrm{ABA}^r_\wedge$ to $\mathrm{ABA}^d_\wedge$



- $Ab^* = \{A^* \mid A \in Ab^\wedge\}$ such that:
  - $\mathcal{L} \cap \hat{A}b = \emptyset$ and
  - whenever $A \neq B$ then $A^* \neq B^*$.
  - Let $\tau(\mathcal{L}) = \mathcal{L}^\wedge \cup Ab^*$.

Definition

Given an **ABF**, $\tau(\textbf{ABF}) = (\tau(\mathcal{L}), \tau(\mathcal{R}), Ab^\wedge, \tilde{\ }, \leq)$ *where:*

- $C \to (\bigwedge_{i=1}^{n} A_i)^* \in \hat{\mathcal{R}}$ *iff:*
  - $A_1, \ldots, A_n \vdash_{\mathcal{R}^\wedge} B$
  - $B \in \overline{C}$
  - $\{A_1, \ldots, A_n\} < C$
- $\tau(\mathcal{R}) = \mathcal{R}^\wedge \cup \hat{\mathcal{R}}$
- *Where* $A \in Ab^\wedge$, *let* $B \in \widetilde{A}$ *iff* $B \in \overline{A} \cup \{A^*\}$.

Example

Let $Ab = \{A, B, F\}$, $\mathcal{R} = \{A, B \to \overline{F}\}$ *and* $A < B < F$. *We have the following translated framework:* $\tau(\mathbf{ABF}) = (\tau(\mathcal{L}), \tau(\mathcal{R}), Ab^{\wedge}, \tilde{\ }, \leq)$ *where:*

- $\tau(\mathcal{L}) = \mathcal{L}^{\wedge} \cup \{A^*, B^*, F^*, (A \wedge B)^*, (B \wedge F)^*, (A \wedge F)^*, (A \wedge B \wedge F)^*\}$.
- $\tau(\mathcal{R}) = \mathcal{R}^{\wedge} \cup \{F \to (A \wedge B)^*\}$.

$\mathrm{ABA}^r_{\wedge}$

- $\{F\} \hookrightarrow_r \{A, B, A \wedge B\}$.
- since $\{A, B\} \vdash_{\mathcal{R}} \overline{F}$ and
- $\{A, B\} < F$.

$\mathrm{ABA}^d_{\wedge}$

- $\{F\} \hookrightarrow_d \{A, B, A \wedge B\}$.
- since $\{F\} \vdash (A \wedge B)^*$ and
- $\{F\} \not< A \wedge B$.

Why new atoms Ab^*?

Example

Let $Ab = \{A, F\}$ *and* $\mathcal{R} = \{A \to F', A' \to D\}$,
$A' \in \overline{A}$, $F' \in \overline{F}$
and $A < F$.

- *Suppose we would add* $F \to A'$ *instead of* $F \to A^*$.
- *Then we would derive information* (D) *not derivable in* ABA^r.

Adequacy

Theorem

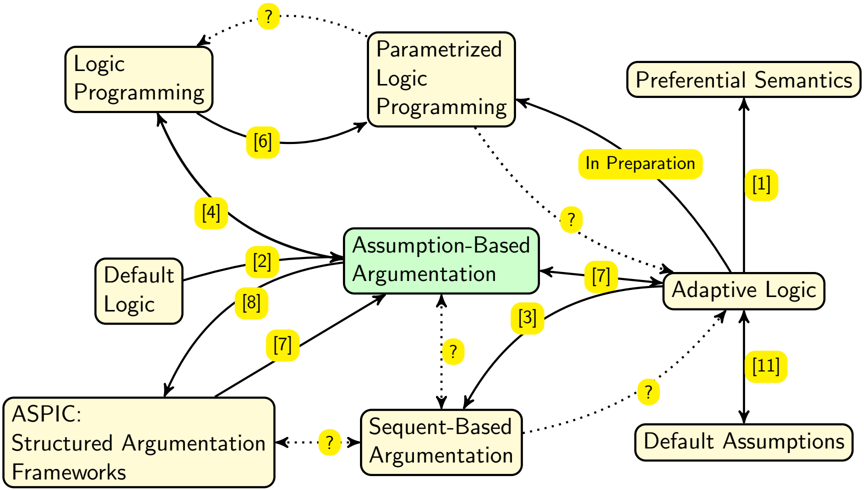
Given an assumption-based framework **ABF***:*
Δ *is r-*$\wedge$*-preferred (in* $\mathbf{ABF}^{\wedge}$*) iff* $\tau(\Delta)$ *is d-*$\wedge$*-preferred (in* $\tau(\mathbf{ABF}^{\wedge})$*)*

Outlook

In the paper we consider . . .:

- Other semantics
- Various lifting principles for non-total orders.
- Various conditions on extensions for non-flat frameworks.

A Broader Picture



Thank you!
Questions or remarks?

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